Probability Distortions Reflect Boundary Repulsions in Noisy Inference

Saurabh Bedi (saurabh.bedi@econ.uzh.ch)
Zurich Center for Neuroeconomics (ZNE), University of Zurich, Switzerland

Gilles de Hollander (gilles.dehollander@econ.uzh.ch)
Zurich Center for Neuroeconomics (ZNE), University of Zurich, Switzerland

Christian C. Ruff (christian.ruff@econ.uzh.ch)
Zurich Center for Neuroeconomics (ZNE), University of Zurich, Switzerland

Abstract

Probability distortions—the apparent overweighting of small probabilities and underweighting of large ones—is central to decision-making under risk, but its normative and mechanistic origins remain unclear. Traditionally seen as irrational, we propose that probability distortion instead emerges from optimal but noisy inference on bounded quantities. In our proposed account, repulsions arise at natural boundaries of probabilities (0 and 1) due to both resource-rational efficient encoding and Bayesian optimal decoding. Our account predicts that experimental manipulations of boundaries and noise should systematically reshape both probability distortions and behavioral variability, in both risky choice and probability perception. We confirm these predictions in three pre-registered experiments. Our findings reframe probability distortion as a normative consequence of bounded noisy inference and offer a unified mechanistic explanation for its presence across valuation and perception.

Keywords: Probability weighting, Bayesian inference, Efficient coding, Risk, Perception

Theory and Background

In decision-making under risk, people systematically overweight small probabilities and underweight large ones. This pattern of probability distortions, central to Prospect Theory (Kahneman & Tversky, 1979), explains puzzling reversals in risk preferences that are traditionally viewed as irrational. Yet similar distortions arise in tasks involving complexity without risk (Oprea, 2024) and even in simple perceptual judgments (Varey et al., 1980), suggesting a more general underlying mechanism. We propose that probability distortions arise from noisy inference of bounded quantities. When probabilities—bounded between 0 and 1—are inferred under cognitive noise, boundary repulsions emerge from two fundamental principles: 1) resource-limited efficient encoding truncates likelihoods, and (2) Bayesian optimal decoding with bounded priors truncates posteriors. These boundary repulsions generate the classical distorted probability weighting pattern (Fig. 1b, c). Our account makes distinct predictions: 1) New contextual boundaries should create new distortions; 2) Increasing cognitive noise should amplify probability distortions; 3) Variability should decrease near boundaries; 4) Distortions generalize beyond risky decisions to simple perception.

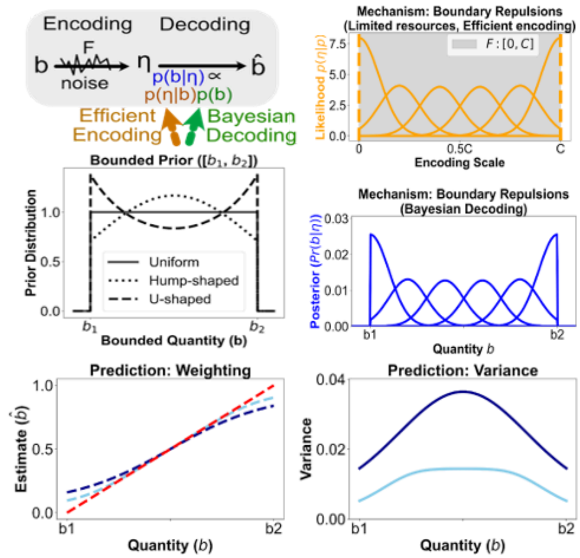

Figure 1: *Boundary repulsions arise in noisy inference due to both efficient encoding and Bayesian optimal decoding of bounded quantities.*

Results

We tested our predictions across three pre-registered within-subject experiments, manipulating probability boundaries (explicit range instructions) and cognitive noise (fraction complexity) in both risky lottery valuation and perceptual probability estimation tasks (Fig. 2). All analyses used pre-registered generalized linear mixed-effects models on data from pre-registered bins near boundaries.

To test predicted effect of cognitive noise, we modeled responses as (*response ~ 1 + probability + range × noise + (1|subject))*. Within bins near the natural boundaries (0 and 1), higher noise indeed increased responses (overweighting) above 0 and decreased responses (underweighting) below 1 across tasks ($p < .001$). To test boundary effects, we analyzed behavior around experimentally induced boundaries at 0.5 (Experiment 1) and 0.34/0.66 (Experiments 2 and 3). As predicted, responses decreased just below and increased just above boundaries, producing "double" and "triple" distortion patterns ($p < .001$) (Fig. 3).

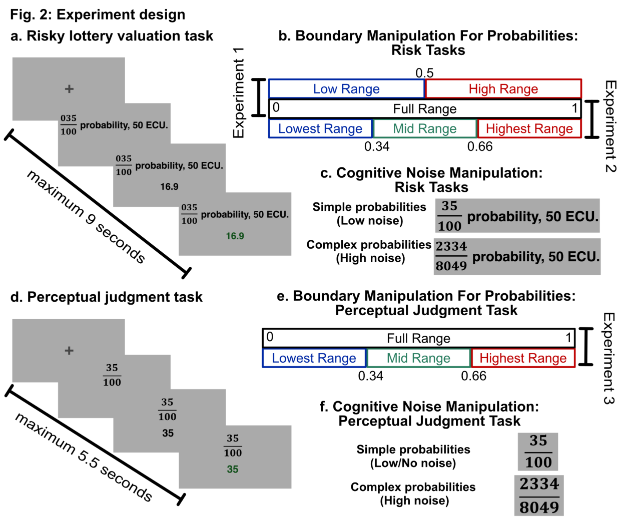

Figure 2: *Experimental design involving risky lotteries and perceptual probability estimation.*

Finally, we confirmed that variability (*var ~ 1 + probability + range × noise + (1|subject)*) indeed decreased near boundaries ($p < .001$). Therefore, all our hypotheses were confirmed across all experiments (n = 71, n = 69 for risky valuation; n = 59 for perceptual estimation).

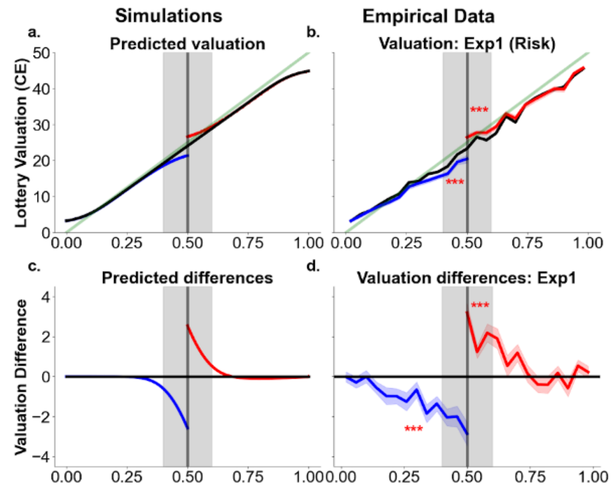

Figure 3: *Adding a contextual boundary at 0.5 creates a double weighting function, shifting lottery valuations by ~15% within subjects.*

Discussion

By linking probability distortions to Bayesian optimal decoding and resource-rational efficient encoding, our account explains them as a normative consequence of noisy inference on bounded quantities. This account makes several distinct predictions that we empirically confirm in pre-registered experiments for risky valuation and perceptual estimation tasks.

Critically, we confirm a unique prediction: distortions are systematically coupled to reductions in behavioral variability near boundaries—an effect existing noisy inference models (Enke & Graeber, 2023; Frydman & Jin, 2023; Khaw et al., 2022; Zhang & Maloney, 2020) cannot explain. These models rely on specific structural assumptions (e.g., log-odds transformations, fixed anchors, specific priors) that introduce degrees of freedom but do not predict these coupled shifts of distortions and variance.

In contrast, our account shows how boundary repulsions arise from the interaction of cognitive noise and boundedness, providing a parsimonious, mechanistic, and general account of probability weighting across domains.

References

Enke, B., & Graeber, T. (2023). Cognitive uncertainty. *The Quarterly Journal of Economics*, *138*(4), 2021-2067.

Frydman, C., & Jin, L. J. (2022). Efficient coding and risky choice. *The Quarterly Journal of Economics*, *137*(1), 161-213.

Kahneman, D., & Tversky, A. (1979). Prospect Theory: An Analysis of Decision under Risk. *Econometrica*, *47*(2), 263–291. https://doi.org/10.2307/1914185

Khaw, M. W., Li, Z., & Woodford, M. (2022). *Cognitive imprecision and stake-dependent risk attitudes* (No. w30417). National Bureau of Economic Research.

Oprea, R. (2024). Decisions under risk are decisions under complexity. *American Economic Review*, *114*(12), 3789-3811.

Varey, C. A., Mellers, B. A., & Birnbaum, M. H. (1990). Judgments of proportions. *Journal of Experimental Psychology: Human Perception and Performance*, *16*(3), 613.

Zhang, H., & Maloney, L. T. (2012). Ubiquitous log odds: a common representation of probability and frequency distortion in perception, action, and cognition. *Frontiers in neuroscience*, *6*, 1.