A Hierarchical Multivariate Copula based Framework for Cognitive Modeling

Jesper F. Ehmsen (Jesperfischer@cfin.au.dk)
Aarhus University, Denmark

Arthur S. Courtin (@cfin.au.dk)
Aarhus University, Denmark

Francesca Fardo (@cfin.au.dk)
Aarhus University, Denmark

Abstract

Computational cognitive models provide an approach to understanding behavior and cognition by formalizing latent parameters underlying decision-making and learning. Many existing models take a univariate approach, analyzing single measures in isolation, while others incorporate multiple measures but impose specific process assumptions that constrain how these measures relate i.e. drift diffusion models. Here, we introduce a hierarchical multivariate modeling framework that uses copulas to flexibly combine independent likelihood functions, enabling joint modeling of multiple measures without imposing restrictive assumptions. Through simulations and empirical applications, we assess the reliability, discriminability, and advantages of copula-based modeling (CBM). Model validation via simulation-based calibration, model recovery, and sensitivity analyses demonstrate that CBM is computationally robust and accurately recovers latent parameters and their uncertainty. When applied to psychophysical and probabilistic learning tasks, CBM can be empirically distinguished from DDMs, even with limited data. We show that this framework enables efficient use of available data by integrating multiple sources of information, while enhancing model accuracy and efficiency of parameter estimation.

Keywords: Copula; Multivariate; Hierarchical; Framework

Introduction

Cognition spans a wide spectrum of interacting processes, many of which remain only partially understood. The challenge of unraveling these processes lies both in their quantification but also in developing models that accommodate the richness of cognitive phenomena. Traditional approaches often rely on univariate analyses that isolate individual measures, such as choices, response times or physiological signals. This approach risks discarding meaningful dependencies between variables and overlooking the interactions that could shape behavior and cognition.

Computational cognitive models offer a principled framework for inference about cognitive processes by formalizing latent parameters that govern decision-making. A particularly influential class of such models, Drift Diffusion Models (DDM), posits that decision-making arises from an evidence accumulation process, in which noisy information accrues over time until a decision threshold is reached (Ratcliff, 1978). The DDM's ability to jointly model binary choices and response times has made it a cornerstone of cognitive modeling. However, its reliance on an accumulation-to-bound mechanism imposes a specific process assumption that may not be suitable for all cognitive tasks. We introduce Copula Based Modelling (CBM), a hierarchical multivariate modeling framework that uses copulas to flexibly model dependencies between multiple measures. Our framework decouples marginal distributions from dependency structures with the use of copulas, which means that CBM enables joint modeling of multiple measures without restrictive assumptions about their relationships. This flexibility makes CBM applicable to a broad range of scientific questions.

Methods

CBM builds on three steps: (1) each outcome (e.g., choice, RT) is modeled using a domain-specific likelihood function; (2) experimentally manipulated variables or predictors in general are mapped to the outcomes through latent parameters; (3) dependencies among outcomes are modeled via a copula, which captures correlation beyond shared predictors. We built two CBMs based on the assumption that response times and response probabilities are linked by entropy.

We here focus on binary choices and response times, to enable comparison with the DDM. We evaluated our CBMs using simulations and ensured they were distinguishable from the DDM and fit both types of models into two public datasets: an orientation detection task (Bang et al., 2019) and a probabilistic reward-learning task (Hess et al., 2024). For comparison, we fit DDMs with matched priors and inference settings. Models were implemented in Stan, and comparisons used leave-one-out cross-validation (LOO-CV).

Our two formulated CBM models had binary choices following a Bernoulli likelihood and response times a shifted log-normal likelihood with an additional non-decision parameter. The expectation (E_t) governing the random variable of the Bernoulli distribution was determined by a psychometric function or a Rescorla Wagner learning model. Mean response time was then assumed to be generated from an affine function of the entropy of this expectation.

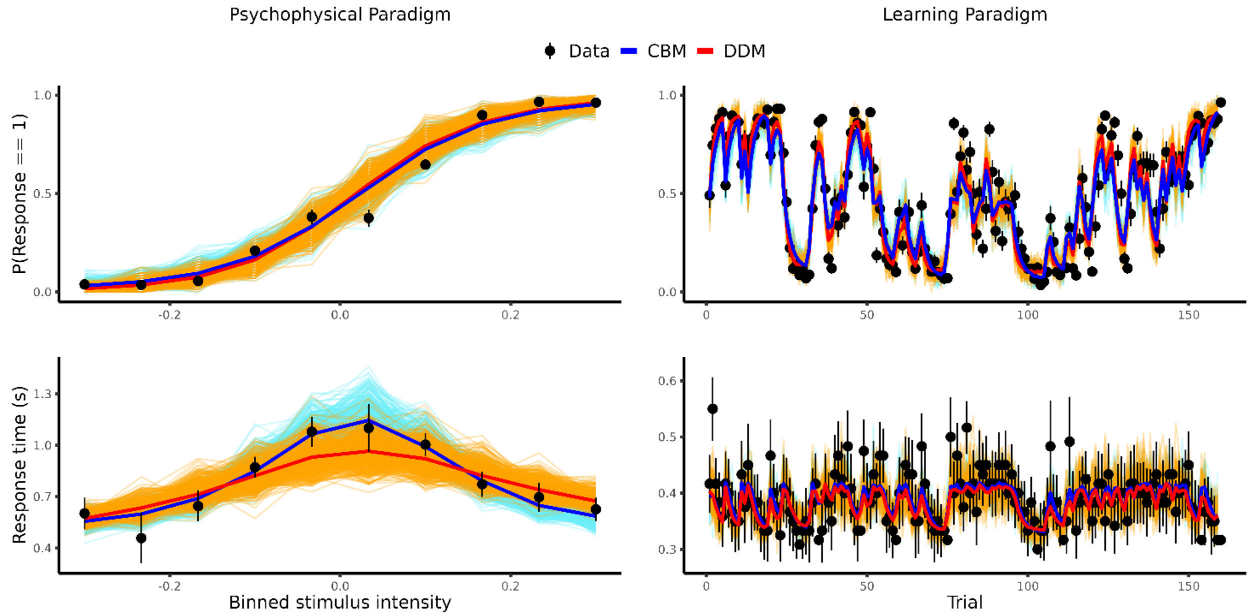

Figure 1. Posterior predictive checks for experimental data from a psychometric paradigm (left) and a learning paradigm (right). Group-averaged data (black dots with 95% confidence interval). The top row depicts the probability of a response, while the bottom row presents response times. Marginal posterior predictive means and medians are shown for both the drift diffusion model (DDM) (red) and the multivariate hierarchical copula-based framework (CBM) (blue). Orange and light blue lines represent posterior predictive draws for DDM and CBM.

$$\widehat{RT}_t = RT_{int} + RT_{slope} \cdot H(E_t)$$

where $\widehat{RT}_t$ is the expected response time at trial t, RT_{int} reflects the response speed under minimal uncertainty, $H(E_t)$ is the degree of uncertainty in an expectation and RT_{slope} quantifies the efficiency with which subjects resolve uncertainty during decision-making, measured in seconds per bit. To model the dependency between the binary choices (B_t) and response times (RT_t) above their mutual dependence of the expectation (E_t) induced by the experimentally manipulated variables X_t (stimulus strength and reward), we apply the probability integral transform to then transform these into a multivariate normal (MVN) distribution using is the normal quantile function $\Phi^{-1}(.)$.

$$\begin{bmatrix} \Phi^{-1}(F_1(B_t \mid X_t)) \\ \Phi^{-1}(F_2(RT_t \mid X_t)) \end{bmatrix} \sim MVN\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}\right)$$

Where F_1 and F_2 are the cumulative distribution functions of each measure, and ρ represents the correlation between binary choices and response times, beyond what is explained by X_t.

The correlation parameter ρ captures linear dependence on the transformed marginal space. This parameter quantifies residual associations between response times and choices beyond their marginal distributions. Positive values indicating that longer RTs are associated with higher likelihood of choosing “1”. Additionally, the copula’s negative entropy provides a lower bound on mutual information, serving as a diagnostic for dependencies that are not captured by the assumed relationship and specified marginals.

Results

CBM demonstrated reliable simulation-based calibration and parameter recovery on the group mean parameters. Indicating good parameter estimation and well calibrated credibility intervals. Model recovery analyses showed that the CBM was distinguishable from DDMs in over 99% of cases. Indicating that if one of the two models generated the data then we can be quite confident that we can pick up that difference. This model comparison also entailed a reduced model with only binary choices to show that the inclusion of response times led to a reduced uncertainty in parameter estimates across tasks confirming that RTs provide complementary information, both in reducing uncertainty in parameter estimates. Lastly, we fit the DDM and the CPM to two public datasets (Figure 1). Here we found that CBM provided better fit than the DDM when considering both choices and RTs (elpd difference: psychophysics = -153 ± 17; learning = -554 ± 48).

References

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