Approximate Bayesian computation with a complex internal model naturally combines probabilistic inference and heuristics

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Abstract

Proposals differ on how the brain accounts for the uncertainty of perceptual variables – either by representing them as probability distributions that explicitly encode uncertainty in their width (Knill & Pouget, 2004), or by exploiting the correlation between the uncertainty of one variable (e.g., orientation) and the value of others (e.g., contrast), using the latter's point estimates as heuristic proxies (Bertana, Chetverikov, van Bergen, Ling, & Jehee, 2021). The two approaches offer distinct advantages – probabilistic representations provide superior data- and memory-efficiency, while proxy-based strategies impose substantially lower computational demands – and each has its proponents, depending on which advantage is considered more relevant to brain function (Barthelmé & Mamassian, 2010; Meyniel, Sigman, & Mainen, 2015; Koblinger, Fiser, & Lengyel, 2021). Rather than strictly contrasting these hypotheses, we follow a normative perspective and argue that both strategies can emerge naturally in a unified framework when time-evolving approximate inference is optimized to solve realistic tasks involving the joint estimation of multiple interacting variables. We formalize this idea by modeling behavior as the output of an ideal observer that combines approximate probabilistic perceptual representations with fast, coarse proxy information – yielding a flexible hybrid approach. Through simulations, we show that the model adaptively relies on proxies to compensate for the coarseness of approximate inference. Finally, by directly comparing the model's output to empirical data, we demonstrate that observed behavior qualitatively aligns with the predictions of this hybrid model.

Keywords: probabilistic models; Bayesian perception; heuristics; proxy

Introduction

Traditional experimental approaches often frame probabilistic and proxy-based models as a strict dichotomy (Barthelmé & Mamassian, 2010), and try to distinguish between them by designing tasks that heavily capitalize on the hallmark advantages of probabilistic inference – such as data and memory efficiency – where non-probabilistic models are expected to fail (Körding & Wolpert, 2004; Maloney & Mamassian, 2009). In contrast, to explore the potential hybrid solutions, we adopted a fundamentally different approach – shifting the focus from absolute performance to the behavioral signatures of plausible algorithmic realizations.

We built the hybrid probabilistic model around three key considerations: first, navigating environments of realistic complexity requires the automatic inference of multiple interacting variables (Koblinger et al., 2021) – many of which can serve as proxies depending on the decision context; second, complex probabilistic inference must rely on approximations to remain tractable; and third, most of the biologically plausible approximation mechanisms unfold over time. This time dependence leads to a fundamental prediction: the quality of the probabilistic approximation improves over time, making the subjective uncertainty of the observers increasingly predictive of their behavioral accuracy (Lengyel, Koblinger, Popović, & Fiser, 2015). We refer to this measurable behavioral signature as the calibration of uncertainty. While most process-level perceptual models predict improved accuracy over time, a corresponding improvement in calibration is specific to probabilistic methods.

Our framework also incorporates the possibility of proxy-based shortcuts, which provide rapid – nearly instantaneous – uncertainty estimates based on crude point estimates of the complex model's proxy variables available early in their automatic inference. Crucially, the degree to which behavior relies on proxies depends on two factors: the reliability of the proxies and the quality of the probabilistic inference of the primary variable – the latter improving with time.

Model simulations

We simulated behavior in a hypothetical orientation estimation task using stimuli of varying contrast, modeling perception and behavioral judgments as separate processes (Fig. 1A). Orientation perception was implemented using a probabilistic sampling algorithm – a temporally unfolding process that approximates probability distributions via histograms of accumulated representative samples (Fiser, Berkes, Orbán, & Lengyel, 2010). An ideal observer then converted these histograms into orientation estimates and uncertainty judgments (Fig.1A, arrow 1), with the latter potentially informed by the observed contrast (stimulus strength), which served as a proxy for uncertainty (Fig. 1A, arrow 2).

We ran simulations to test the time-dependence (sample size-dependence) of uncertainty calibration (Fig. 1B), quantified as the slope of best-fitting lines through uncertainty–accuracy pairs (with accuracy being the average cosine error), measured at fixed contrast levels. A slope of 45° indicates perfect calibration; a vertical line reflects none. We compared three model variants: one estimating uncertainty from perceptual samples alone (sampling-only), one using only the crude proxies (proxy-only), and a hybrid model combining both sources. Only the sampling-only and hybrid models showed improvement in calibration with increasing sample size. At the single-sample limit (brightest purple lines), the sampling-only model was uncalibrated, while the hybrid model was already calibrated by leveraging proxy information, which it further refined as more samples became available.

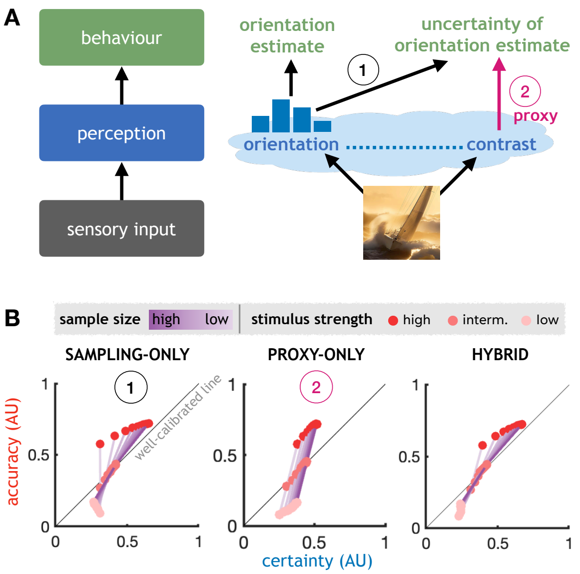

Figure 1: **A**. Overview of the modeling framework. **B**. Simulated calibration as a function of sample size, quantified as the slope of best-fitting lines (purple lines) through accuracy–certainty pairs (red dots) computed at different stimulus strengths.

Experimental results

To assess the nature of human perceptual uncertainty representation, we compared simulation outcomes to human data collected in a novel orientation estimation task (Fig. 2A). In each trial, participants briefly viewed a complex stimulus consisting of multiple randomly oriented items (line segments, N=6; or Gabor patches, N=4) of varying contrast. After the stimulus disappeared, a single target item was cued, and participants simultaneously reported its perceived orientation and their subjective uncertainty about the reliability of their percept. The quality of the hypothesized approximate inference was manipulated via presentation time, and stimulus strength through set size (number of items) and target contrast.

Because stimulus strength in the experiment was influenced by two independent proxies – unlike in the simulations – we adapted our calibration analysis. Accuracy and uncertainty were computed separately for each unique stimulus, defined by the combination of presentation time, contrast, and set size (Fig. 2B, top row, purple points). To assess time-dependent changes in calibration, best-fitting lines were computed for data points sharing the same presentation time (Fig. 2B, top row, purple lines), and the resulting slopes were plotted as a function of presentation time (Fig. 2B, bottom row).

In the line-segment experiment (Fig. 2B, left side), calibration was already high at the shortest presentation time and showed no significant improvement with additional time (rm ANOVA: $F(3, 15) = 1.56$, $p = 0.241$). To test whether this lack of change reflected a ceiling effect, we repeated the experiment replacing line segments with more ambiguous Gabor patches and reduced the shortest presentation time from 50 to 33 ms (Fig. 2B, right side). While calibration was still relatively strong at brief durations, this modified version revealed a gradual and now significant improvement with time ($F(3, 9) = 12.43$, $p = 0.0015$). This improvement is a hallmark of approximate probabilistic computation, while the strong calibration at short durations suggests that participants may either access multiple samples early on or rely on proxies to support inference. Together, these results support the probabilistic account of perception and align with a hybrid inference strategy that integrates proxy-based information.

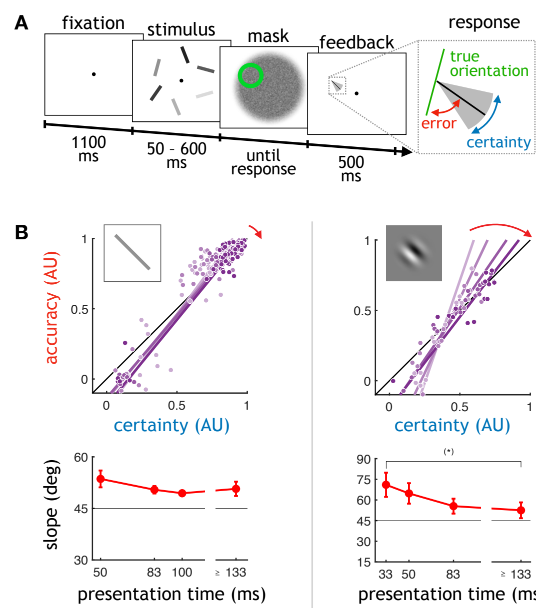

Figure 2: **A**. Trial sequence and behavioral reports (inset). **B**. Empirical time course of uncertainty calibration (darker purple indicates longer presentation times). Data points and fitted lines are averaged across participants for display.

Discussion

This work addresses the under-explored question of how the brain implements efficient inference under naturalistic constraints. By introducing a novel method for identifying behavioral signatures of the underlying computation, we provide evidence that 1) the brain engages in approximate probabilistic inference, and 2) due to the temporarily evolving nature of this inference and reliance on a complex internal model, features are mimicking heuristic computation naturally emerge during such inferences. Our results show that behavior is consistent with inference over complex internal models, in which the brain may leverage proxy-based shortcuts to estimate uncertainty – enabling a flexible hybrid strategy that balances speed and accuracy.

Acknowledgments

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