How neurocomputational mechanisms of number perception adapt to prior expectations

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Abstract

Efficient coding offers a normative theory for how the brain should allocate resources to represent the world, and growing evidence demonstrates that perceptual systems of humans and animals adhere to its principles. However, most existing studies have focused on simple stimuli like Gabor patches and have assumed relatively fixed encoding functions. Here we demonstrate that cognitive and neural representations of numerosity - a higher-level cognitive function - can rapidly adapt to context in ways consistent with efficient coding. Using fMRI (n=39), we show that neural populations tuned to specific numerosities shift their tuning with context, aligning with with a Thurstonian perceptual model in which part of an unconstrained objective stimulus space is linearly mapped to a constrained representational space. Our findings demonstrate how the brain adapts to changing conditions and how neurocomputational modeling of fMRI data can deepen our understanding of the neural representations driving behavior.

Keywords: numerosity; perception; encoding modes; fMRI

Introduction

Because of its limited capacity for storing and processing information, the brain constantly needs to adapt the way it represents the outside world. Intuitively, it should put more resources into representing those stimulus features –and parts of stimulus feature space– that are more likely and/or relevant to the task at hand. A formal, normative framework for this intuition is offered by efficient coding theory (Barlow, 1961). Empirical evidence for efficient coding is rapidly growing and can explain several puzzling behavioral biases (Wei and Stocker, 2015; Prat-Carrabin and Woodford, 2021, 2022). Moreover, perceptual adaptations to the environment are evident in neural processing in early sensory areas, as confirmed with electrophysiological recordings in animals (Grujic et al., 2022).

Behavioral hallmarks of efficient coding are evident not just for simple stimuli but also more abstract stimulus features, such as numerosity (Prat-Carrabin and Woodford, 2022). Also for these features, behavioral data suggest that neural coding can rapidly adapt to changing environmental priors (Prat-Carrabin and Woodford, 2024). However, the neural mechanisms underlying these rapid reconfigurations of numerosity representations are not well-understood.

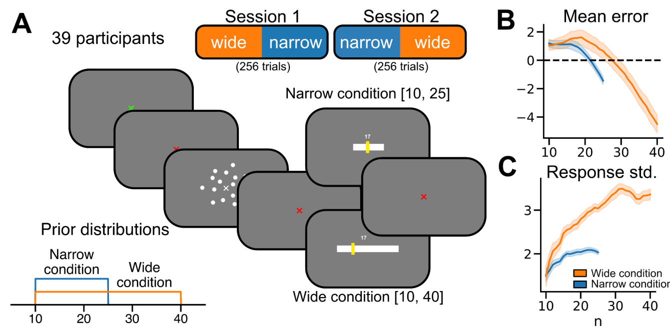

Figure 1. ***A)*** *Overview of task design.* ***B)*** *Mean error for different numerosities and conditions (shaded area is S.E.M.* ***C)*** *Average standard deviation of responses for same numerosities.*

Here, we illuminate the neural mechanisms by which the brain flexibly adapts its magnitude representations to task demands, by leveraging neurocomputational models of numerical representations in the parietal lobe, namely *numerical population receptive field models* (nPRFs; Harvey et al., 2013). We hypothesized that just as how visuospatial tuning of visual receptive fields in visual areas can shift their tuning towards covertly attended stimuli (Olshausen et al., 1993; Klein et al., 2014), neural populations in the intraparietal sulcus (IPS) *tuned* to specific numerosities might shift their preferred numerosity along the number line, to linearly map the currently relevant parts of stimulus space to the full internal representational space (Thurstone, 1927), thereby maximizing representational efficiency

Results

Thirty-nine participants attended two scanning sessions (3T fMRI at 2.5mm resolution) and were instructed to estimate the number of dots in a stimulus area, under two conditions: one with the number of dots always ranging from 10 to 25 ('narrow condition'), and another from 10 to 40 ('wide condition'; see Fig. 1A). Participants received extensive instructions and practiced with feedback at the start of each block.

We tested for behavioral signs of efficient remapping, expecting greater variability and larger biases for perception of the same numbers when viewed in the context with the large number range. Indeed, for numerosities 10-25, participants made larger errors and showed increased variability in the wide condition compared to the narrow condition (errors: $t(38)=4.12$, $p<0.001$; variability: $t(38)=7.27$, $p<0.0001$; Fig 1B, 1C).

We fitted a numerical population receptive field with log-Gaussian receptive fields assuming separate μ (*preferred numerosity*)-parameters for the *narrow* and *wide* conditions, keeping RF dispersion, amplitude and baseline constant. We replicated earlier work, with robust non-monotonic tuning to numerosities around the intraparietal sulcus and postcentral gyrus, in particular in the right hemisphere (see Fig. 2).

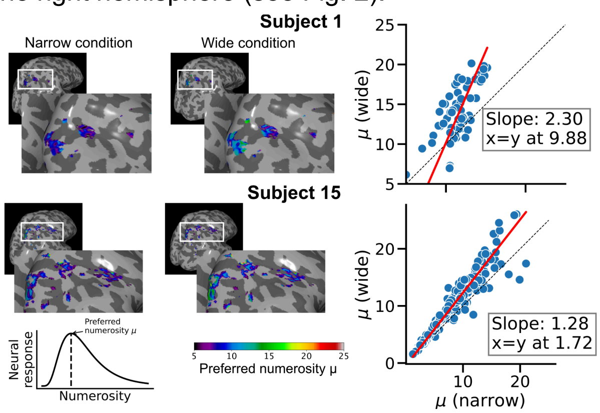

Figure 2. *Example nPRF fits for two representative subjects. Note how small patches of cortex are robustly tuned to specific numerosities (left panels) and how both subjects show a consistent shift in tuning from the narrow to the wide condition.*

To formally test whether numerical tuning shifted between conditions, we compared four models using cross-validated voxelwise R^2 values (cvR^2) within the right IPS. The *no-shift* model assumed constant parameters across conditions. The *free shift* model allowed μ to vary between conditions and was independently fit across voxels. The *efficient* model assumed that the nRFs shifted linearly with slope 2, aligning with the corresponding quantiles of the prior distribution (so $\mu_{\text{narrow}} = \mu_{\text{wide}}$ when $\mu_{\text{narrow}} = 10$). Similarly, the *free slope* model forced all voxels to have the same slope between μ_{narrow} and μ_{wide}, intersecting at 10, but this slope was a free hyperparameter (see Fig. 3A).

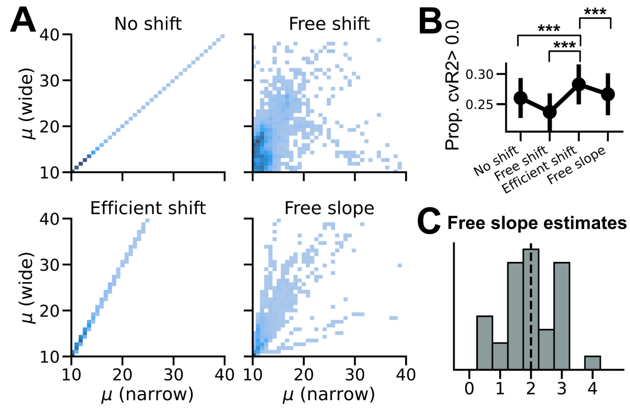

Figure 3. A) *2D histograms of estimated preferred numerosities for narrow and wide conditions, for the four distinct models.* ***B)*** *Proportion of voxels with a cvR2>0.0 within rIPS for the four models. *** p<0.001* ***C)*** *Distribution of estimated slope $\mu_{\text{wide}} \sim \mu_{\text{narrow}}$.*

The number of voxels that showed cvR2 higher than 0 was significantly larger for the *efficient slope*-model compared to all other models (paired t-test, all $p<0.0001$; Fig 2B). The *free slope*-model performed 2nd best. Moreover, the estimated slopes of the *free slope*-model were significantly larger than 1 ($t(38)=5.42$, $p<0.001$) and not significantly different from 2 ($t(38)=5.42$, $p=0.142$; $BF_{01}=2.07$).

Conclusion

Our behavioral and neurocomputational findings show that the human brain can rapidly adapt the processes underlying number perception to changing contexts, by remapping numerical tuning functions in parietal cortex. This remapping provides a concrete neurocomputational mechanism bridging a century-old psychophysics model (Thurstone, 1927) and neurocomputational models of visual attention (Olshausen et al., 1993) and efficient coding (Barlow, 1961). In future work, we will further integrate these cognitive and neurocomputational models.

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