# Efficient Regularization of High-Dimensional Cerebellar Representations by Sparse Parallel Fiber Inputs: A Virtual Sample-based L2 Regularization Perspective

**Jundong Kim (kimjd@gachon.ac.kr)[1]**
**Taerim Yun (lowlyheart@gachon.ac.kr)[1]**
**Chang-Eop Kim (eopchang@gachon.ac.kr)[1*]**

[1] Department of Physiology, College of Korean Medicine, Gachon University, 1342 Seongnam-daero Seongnam Gyeonggi 13120 Republic of Korea

## Abstract

The cerebellum generates extremely high-dimensional representations through parallel fibers (PF) originating from the granule cell layer, enabling precise motor learning and predictive control. However, this excessive dimensionality expansion potentially risks overfitting due to surpassing the intrinsic dimensionality of the input data. This study proposes that the spontaneous and highly sparse PF inputs serve as explicit virtual data samples that effectively implement an L2 regularization mechanism analogous to multiple linear regression. Specifically, the sparse PF inputs mathematically resemble virtual samples, each having a single feature with value $\sqrt{\lambda}$ and target output *y=0*. Improper activation of Purkinje cells (PC) by these sparse inputs triggers error signals via climbing fibers (CF), consequently inducing long-term depression (LTD) at PF-PC synapses. This interpretation extends traditional adaptive filter theories based on the delta learning rule and integrates recent perspectives of spontaneous activity-driven pruning in generative models. Experimental validations using optogenetics and electrophysiology are proposed.



**Keywords:** Cerebellum; Expansion recoding; Parallel fiber; Purkinje cells; Regularization


## Introduction

The cerebellum uniquely expands input signals into extremely high-dimensional spaces. Individual Purkinje cells receive convergent input from over 100,000 parallel fibers, dramatically increasing the dimensionality of representations, facilitating complex motor learning and prediction(Hoxha et al., 2016). However, excessive dimensionality expansion naturally introduces the risk of overfitting. It remains unclear how the cerebellum mitigates this issue. Here, we propose a novel mechanism wherein the naturally sparse spontaneous parallel fiber inputs function as internal L2 regularization agents (**figure 1**).

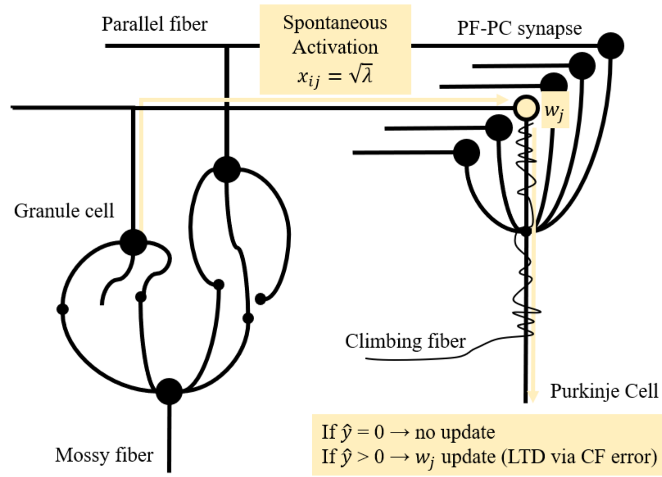



**figure 1.** Spontaneous PF Inputs as Implicit L2 Regularizers in the Cerebellar Circuit

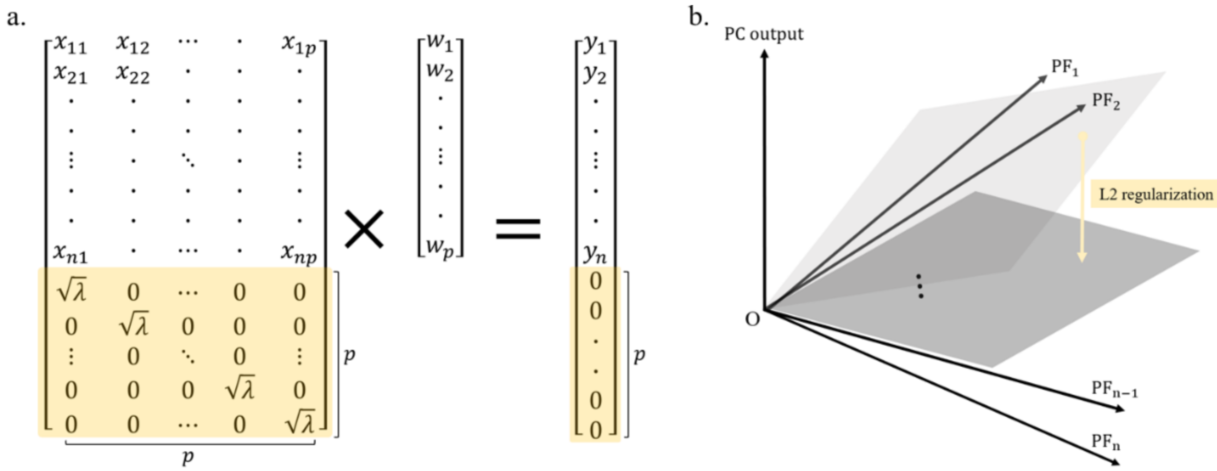
## Theoretical Background

Traditional adaptive filter models suggest that the cerebellum utilizes a delta learning rule, adjusting synaptic weights based on error signals received through climbing fibers(Dean et al., 2010; Fujita, 1982). However, the explicit functional role of PF inputs themselves has remained unclear. We propose redefining the role of sparse PF inputs based on a mathematical equivalence to the L2 regularization in multiple linear regression:

$$\lambda \sum_{i=1}^{p} w_i^2 = \sum_{i=1}^{p} \left( y_i^{(virtual)} - \sum_{j=1}^{p} x_{ij}^{(virtual)} \mathrm{w}_j \right)^2$$

$$= \sum_{j=1}^{p} \left(0 - \sqrt{\lambda} w_j\right)^2$$

Here, each virtual sample input $x_{ij}^{(virtual)}$ equals $\sqrt{\lambda}$ only when *i=j* and zero otherwise, with a target output $y_i^{(virtual)} = 0$ (**figure 2**). This precisely corresponds to the sparse spontaneous activation patterns of PF inputs observed biologically.




## Sparse PF Inputs as Virtual Samples

Biologically plausible estimates indicate that parallel fibers fire spontaneously at ~0.28 Hz per fiber, so fewer than ~0.03 % of the total PF population is active at any given instant, in line with the virtual-sample framework(Steuber et al., 2007). Purkinje cells ideally should not respond ($\hat{y}=0$) to these sparse inputs; inappropriate activation ($\hat{y}>0$) generates explicit CF error signals, thereby weakening the corresponding PF-PC synapses through LTD (**figure 1**). This is precisely aligned with the predictions of the delta learning rule, indicating that sparse spontaneous PF inputs naturally implement internal L2 regularization within cerebellar circuits.

## Experimental Validation Scenarios

Our theoretical predictions can be specifically validated through the following experiments:
a. Utilize optogenetics to artificially modulate the spontaneous activation frequency of parallel fibers and employ electrophysiological techniques to record responses from Purkinje cells and CF error signals.
b. Systematically vary the sparsity and frequency of PF inputs and observe subsequent long-term structural and functional changes at the PF-PC synapses, confirming the theoretical predictions regarding regularization.

## Integrative Interpretation with Existing Studies

Recent studies propose that spontaneous neural activity in generative models leads to pruning of unnecessary connections(Pezzulo et al., 2021). Our proposal expands this idea, suggesting that spontaneous cerebellar PF activation explicitly serves as L2 regularization to prune synaptic connections effectively. This model further relates to synaptic pruning observed during developmental stages and sleep, providing a generalized and integrated framework for understanding cerebellar function.

## Conclusion and Implications

This study theoretically establishes sparse cerebellar PF inputs as efficient intrinsic mechanisms for preventing overfitting in high-dimensional representational spaces. The proposed mechanism and experimental validations offer significant insights into cerebellar functional efficiency and robustness, potentially guiding future neuroscience and artificial intelligence research directions.

## Acknowledgments

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