Up-down states and criticality in a chain of adaptive excitable integrators

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Abstract

We describe a chain of unidirectionally coupled adaptive excitable elements slowly driven by a stochastic process from one end and open at the other end, as a minimal toy model of unresolved irreducible uncertainty in a system performing inference through a hierarchical model. Threshold potentials adapt slowly to ensure sensitivity without being wasteful. Activity and energy are released as intermittent avalanches of pulses with a discrete scaling distribution largely independent of the exogenous input form. Subthreshold bistability closely resembles empirical measurements of intracellular membrane potential. We suggest that critical cortical cascades emerge from a trade-off between metabolic power consumption and performance requirements in a critical world.

Keywords: hierarchical modeling; diffusion; self-organization

We surmise that criticality in the brain ensues from three related preconditions: (1) any living creature, insofar as it comprises a regulator (nervous system) selected to enhance survival odds, must have a good enough model of its environment or external milieu, (2) the multilayered architecture of the brain is a reflection of the multiple spatio-temporal scales of its environment, and (3) a balance between performance in terms of representational accuracy and metabolic power consumption is a characteristic of life indispensable not to overstep the limited homeostatic range compatible with survival.

The perfusive cascade model simulates the behavior of a good regulator that incorporates a dynamical model of its environment. Let there be $l = 1..n_l$ threshold integrator units or levels, coupled in a daisy chain with linear topology. The first and fastest unit is driven by an exogenous input at discrete time steps t = 1, ..., nt . We will assume the input to be distributed as a discrete white standard Gaussian noise process $I \sim N(0, 1)$. This is justified by the homeostatic equilibrium in which living creatures coexist with their environment.

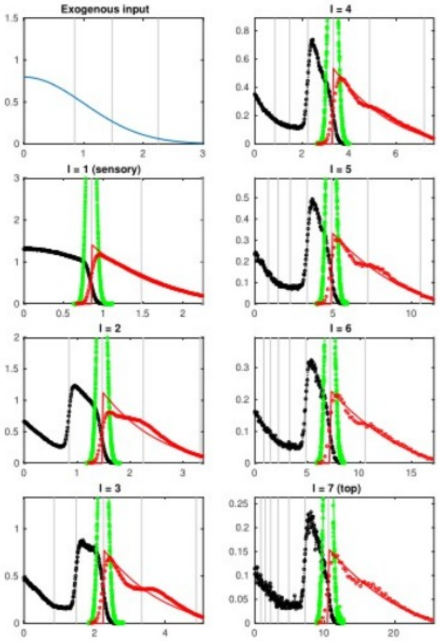

Figure 1: Threshold (green dotted), subthreshold activity (black dotted), and pulse (red dotted) numerical distributions for l = 1..7, and white Gaussian noise exogenous input (blue). All distributions are symmetric with respect to zero. Only the positive half of the (symmetric) densities is shown. The subthreshold activity is shown excluding the rest state at zero.

The system operates in a stop-and-go manner: exogenous input drives the sensory unit only after the chain has reached quiescence. At each time step or iteration t, the exogenous input drives the sensory unit (l = 1), and in general any unit l may receive a signal ε_l-1 from its subordinate neighbor l − 1 and add to its sub-threshold activity $a_l \in R$, yielding the post-pulse activity

$$\tilde{a}_l^{(t)} = a_l^{(t)} + \varepsilon_{l-1}^{(t)}.$$

The unit then fires and resets (like the integrate-and-fire neuron), or stays unchanged according to the firing rule

$$a_l^{(t+1)} = \begin{cases} \tilde{a}_l^{(t)} & \text{if} \quad |\tilde{a}_l^{(t)}| < \theta_l^{(t)} \quad \text{(rest)} \\ \alpha \tilde{a}_l^{(t)} & \text{else.} \qquad\qquad\quad \text{(fire, reset)} \end{cases}$$

where $\varepsilon_l \in R$ is the error signal or pulse, which is propagated forward between consecutive units whenever activity reaches the threshold $\theta_l \in R^+$ (for the sensory unit the error signal is the exogenous input ε_0 = I). Error signals, pertaining to the subsequent iteration t+1, are computed after updating the activities

$$\varepsilon_l^{(t+1)} = \begin{cases} I & \text{if} \quad l = 0 \\ 0 & \text{if} \quad l > 0 \text{ and } \tilde{a}_l^{(t)} < \theta_l^{(t)} \\ \tilde{a}_l^{(t)} & \text{if} \quad l > 0 \text{ and } \tilde{a}_l^{(t)} \geq \theta_l^{(t)}. \end{cases}$$

The thresholds θ_l are gates that set the value of $|a_l|$ beyond which pulses are emitted. Crucially, this mimics the trade-off between metabolic power consumption and performance by setting how much surprisal can be tolerated without (costly) updating the internal representation. θ_l is the time series of threshold fluctuations defined by:

$$\theta_l^{(t+1)} = \begin{cases} \theta_l^{(t)}(1 - w^-) + |\tilde{a}_l^{(t)}| w^- & \text{if } |\tilde{a}_l^{(t)}| < \theta_l^{(t)} \\ \theta_l^{(t)}(1 - w^+) + |\tilde{a}_l^{(t)}| w^+ & \text{if } |\tilde{a}_l^{(t)}| \geq \theta_l^{(t)}. \end{cases}$$

Using white Gaussian noise as driving input, numerical evaluation with L = 7, n_t = 9·106, and w = .01, we obtain the numerical distributions for Al and Θ_l in Fig. 1., and similarly in Fig. 2 for autocorrelated Gaussian input.

Remarkably, the bimodal perfusive cascade subthreshold activity densities (Fig. 2, note the probability mass g associated to zero is not shown) are similar to in vivo intracellular spontaneous membrane potential shift recordings (Fig. 3) after equating zero to resting membrane potential and hΘi to threshold potential. Hyperpolarizing currents lead to near-unimodal densities, whereas depolarizing currents lead to bimodal densities in a manner that strongly resembles the PC densities induced by driving input of Hurst param56eter H < 0.5 and H > 0.5 respectively (Fig. 2).

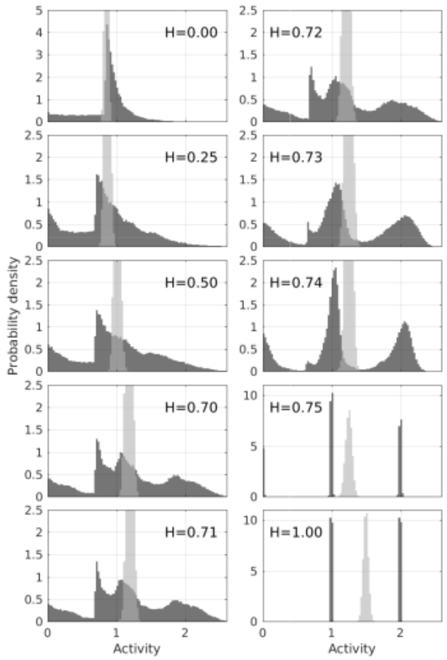

Figure 2: Histogram of positive half densities of post-pulse activity Ã (dark) and threshold Θ (light) for different degrees of Gaussian input autocorrelation (indexed by the Hurst parameter H).

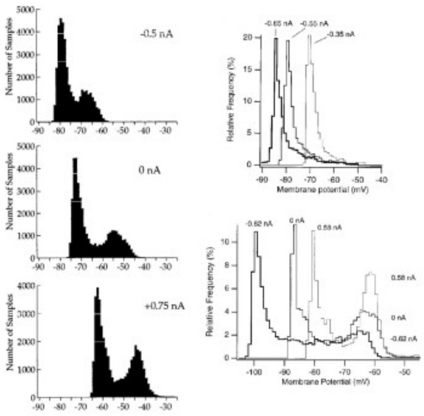

Figure 3: Effect of polarizing current on membrane potential density experimental recordings.
Left: In vivo intracellular recordings of spontaneous activity of neostriatal spiny cells displaying Up-Down modes. Right: Cat deep pyramidal neuron recordings under anesthesia under hyperpolarizing and depolarizing currents. Reused from Wilson & Kawaguchi (1996) and Pare et al. (1998).

References

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