Modeling the Hierarchy of the Human Olfactory Perceptual Space via Hyperbolic Embeddings

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Abstract

Olfactory perception is a complex, high-dimensional process, still largely understudied compared to vision or audition. In this work, we investigate the hierarchical organization of human olfactory perception by embedding perceptual data in hyperbolic space. Hyperbolic geometry, characterized by its exponential volume growth, is particularly well-suited for capturing hierarchical structures. We apply a contrastive learning approach to embed olfactory perceptual data in the Poincaré ball model of hyperbolic space and analyze its structural properties. Our results reveal that odorants with higher perceptual entropy, indicative of greater uncertainty or ambiguity in their perceptual descriptors, tend to be positioned closer to the center of the Poincaré disk, while odorants with lower entropy, reflecting more consistent and distinct perceptual judgments, are mapped toward the boundary. Additionally, individual differences in olfactory perception are reflected in the spatial distribution of embeddings, suggesting that confidence, personality traits, and perceptual biases may influence the way odors are structured in the human olfactory perceptual space. These findings provide a computational framework for modeling olfactory perception. Our approach contributes to the broader goal of understanding the computations underlying sensory perception, bridging cognitive science, neuroscience, and machine learning.

Keywords: Hyperbolic geometry, olfactory perception, representation learning

Introduction

Despite its fundamental role in human perception, **olfaction** remains significantly understudied compared to vision and audition. While substantial progress has been made in characterizing visual and auditory perception through mathematical models and structured representations (Sucholutsky et al., 2023; Brohan et al., 2023; Du, Liu, Li, & Zhao, 2022; Ganis, Thompson, & Kosslyn, 2004; Friederici, 2012), olfaction lacks a comparable theoretical framework. Unlike vision, where color perception has been systematically mapped through the Commission Internationale de l'Éclairage (CIE) color spaces (de l'Éclairage, 1931), or audition, where frequency encoding has been formalized using Fourier space (Evans, 1977), no equivalent mapping exists for the olfactory perceptual space.

Recent evidence strongly suggests that the non-Euclidean **hyperbolic geometry** provides a representation of olfaction that is more biologically accurate as compared to Euclidean models. Notably, (Zhou, Smith, & Sharpee, 2018) demonstrated that olfactory perception is better modeled in a hyperbolic space than a Euclidean one by leveraging Betti curve analysis, a topological data analysis technique that characterizes the structure of high-dimensional datasets. Their findings indicate that olfactory perceptual space exhibits a natural **hierarchical** organization. However, their study focused on hyperbolic embeddings of dimensions higher or equal than three, leaving open the question of whether lower-dimensional hyperbolic spaces, could also capture key structural properties of olfactory perception.

Building upon the above motivation and inspired by (Nickel & Kiela, 2017a), we propose a **contrastive learning** framework to embed olfactory perceptual data in a two-dimensional hyperbolic space. To this end, we leverage the Poincaré ball model of hyperbolic geometry. By applying hyperbolic dimensionality reduction, we can capture the structure of olfactory perception while preserving its intrinsic hierarchical organization. Unlike previous studies that employed three or more dimensions in hyperbolic space (Zhou et al., 2018), we demonstrate that a two-dimensional hyperbolic representation is sufficient to capture key perceptual relationships. We investigate the two-dimensional embedding inferred by our method, and find that **perceptual entropy** correlates with deeper levels in the hyperbolic hierarchy. This type of entropy, which can be understood as a form of olfactory uncertainty, may constitute a novel axis in the perceptual olfactory space, complementing the other fundamental axes identified in previous research (Crocker & Henderson, 1927; Koulakov, Kolterman, Enikolopov, & Rinberg, 2011). This result not only enhances interpretability but also suggests fundamental insights on the cognitive perceptual encoding of odors. Our study represents a step toward a theoretical framework for understanding olfactory perception, bridging theoretical modeling and empirical data analysis in cognitive computational neuroscience.

Dataset

We used a publicly available version of *Sagar* dataset (Sagar, Shanahan, Zelano, Gottfried, & Kahnt, 2023) from the Pyrfume repository (Castro et al., 2022), where perceptual labels of odorants are provided by humans when exposed to odorant stimuli. This dataset contains ratings from 3 human subjects for 160 odorants with respect to 15+3 perceptual descriptors. In addition to 15 common descriptors, there are 3 more descriptors that vary among subjects, which we excluded for coherence. The provided ratings are normalized within the range of $[-1,1]$, so that the dataset has form $\mathcal{D} \subset [-1,1]^{15}$.

Poincaré model

The n-dimensional hyperbolic space is the unique Riemannian manifold of constant curvature equal to -1. The Poincaré ball model of the hyperbolic space is given by the open Euclidean ball $\mathbb{P}^n = \left\{z \in \mathbb{R}^n \mid \|z\|^2 < 1\right\}$, where $\|.\|$ is the Euclidean norm, equipped with the geodesic distance:

$$d_{\mathbb{P}}(z,z') = \operatorname{arccosh}\left(1 + \frac{2\|z-z'\|^2}{(1-\|z\|^2)(1-\|z'\|^2)}\right). \quad (1)$$

Method

Given a dataset $\mathcal{D} \subset \mathbb{R}^n$ in a Euclidean space, the goal is to embed it in the 2-dimensional Poincaré ball by approximately preserving the distances, obtaining $\mathcal{D}' \subset \mathbb{P}^2$. We employed a hyperbolic contrastive embedding procedure inspired

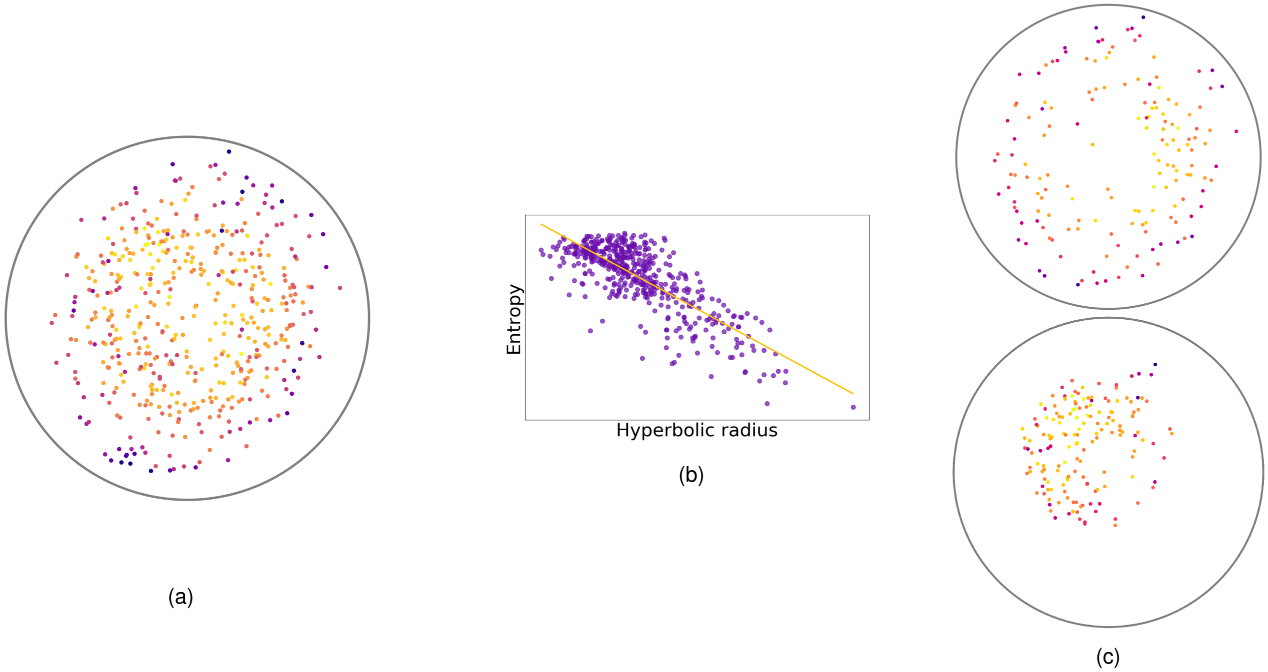

Figure 1: a) Hyperbolic embeddings of Sagar Perceptual dataset. b) Correlation between the entropy and the hyperbolic radius across the embeddings. c) Hyperbolic embeddings of perceptual data for two different participants.

by (Nickel & Kiela, 2017b). This method optimizes a loss forcing neighboring data points ('positive pairs') to be embedded close to each other, with an additional loss term encouraging $\mathcal{D}'$ to be well-distributed. We modified the loss via a metric term forcing the embedding to preserve the distances between positive pairs. The steps of our method are as follows:

- Construct the set $P \subseteq \mathcal{D} \times \mathcal{D}$ of *positive pairs*, defined by the k-nearest neighbor relation, where k is a hyperparameter.

- Initialize the embedding $\mathcal{D}' \subset \mathbb{P}^2$ by sampling i.i.d. from a hyperbolic Gaussian distribution.

- Minimize the contrastive loss $\mathcal{L} = \mathcal{L}_P + \mathcal{L}_N$ via Riemannian Stochastic Gradient Descent on the Poincaré ball (Nickel & Kiela, 2017b). Below we define the terms of the loss $\mathcal{L}$, where d denotes the Euclidean distance in the ambient data space, $d_{\mathbb{P}}$ the Poincaré distance in the embedding space, and γ the *temperature* hyperparameter:

$$\mathcal{L}_P(\mathcal{D}') = \frac{1}{|P|} \sum_{(x,y)\in P} (d_{\mathbb{P}}(x',y') - d(x,y))^2/\gamma$$

$$\mathcal{L}_N(\mathcal{D}') = \frac{1}{|\mathcal{D}|^2 - |P|} \log \sum_{(x,y)\notin P} e^{-d^2_{\mathbb{P}}(x',y')/\gamma}$$

Results

We present the embeddings of the olfactory perceptual space in the Poincaré ball. We set the learning rate to 0.001, $\tau = 0.1$, $k = 20$, and batch size to 240, and trained the model for 1000 epochs.

We employ *entropy* as a measure of hierarchy in the perceptual space. Specifically, a data point $x \in [-1,1]^{15}$ is first normalized via a softmax to a probability distribution over descriptors $p = (p_1, \dots, p_{15})$, whose entropy is then computed as $\mathcal{H}(p) = -\sum_{i=1}^{15} p_i \log p_i$. Intuitively, entropic labels reflect uncertainty by the human subject when evaluating the odorant. Therefore, entropy measures the information content in the perceptual space, defining a hierarchy between more informative and less informative odorants.

Figure 1 (a) visualizes the hyperbolic embeddings, where each data point is colored according to its entropy; cooler colors indicate a higher value. The hierarchy emerges in the embeddings since data points positioned closer to the boundary of the hyperbolic ball exhibit a higher entropy. We also computed the correlation between the entropy and the hyperbolic radius $d_{\mathbb{P}}(x', \mathbf{0})$, $x' \in \mathcal{D}'$, averaged over 10 random seeds, whose value is 0.79 ± 0.02 (see Figure 1 (b) for a correlation plot). These results not only suggest that the olfactory perceptual space may have a hierarchical structure, where hierarchical levels correspond to the degree of certainty in the odor perception, but that this structure clearly emerges when embedding data in a hyperbolic space. Moreover, in Figure 1 (c), different embedding patterns across subjects can be observed. These representations could be investigated further, as they may provide insights into differences in olfactory perception across humans, including olfactory loss or disability.

Acknowledgments

This work has been supported by the Swedish Research Council, Knut and Alice Wallenberg Foundation, and the European Research Council (ERC-2023-SyG 10118977 D2Smell).

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