Testing an Integrative Framework for the Sense of Control

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Abstract

The extent to which we experience a sense of control over our environment shapes how we perceive the world and plan our actions. But when precisely do we consider ourselves to be in control? Here, we define the sense of control as the 'degree of *a priori* readiness to collect rewards that have yet to be announced.' We formalize this notion and propose degree-ℓ empowerment Emp_ℓ as a unified measure that integrates various conceptualizations of the sense of control. ℓ is a free parameter that regulates how the sense of control depends on three fundamental determinants: (i) action availability, (ii) *certain* achievability of potentially desired outcomes, and (iii) *possible* achievability of potentially desired outcomes. We show that Emp_ℓ accurately predicts more than 80% of participants' decisions in an experimental paradigm in which they choose between possible and certain achievability of future rewards, and that the value of ℓ effectively captures inter-individual differences in participants' preferences that are also associated with scores on the widely recognized Locus-of-Control survey. Our findings thus lay a foundation for identifying the human sense of control and investigating its relationships with personality traits, psychological disorders, and broader sociological conditions.

Keywords: Control; Agency; Self Efficacy; Empowerment; Decision Making; Planning; Reward; Intrinsic Motivation

Introduction

The extent of one's sense of control over the environment is central to human behavior (Rotter, 1966; Ajzen, 2002; Leotti, Iyengar, & Ochsner, 2010). It is often agreed that humans consider themselves to be in control when they can reliably achieve desired outcomes; however, there are various approaches for quantifying this concept. One set of measures operationalizes the sense of control through task-specific experimental variables that effectively explain certain aspects of human behavior (Dorfman & Gershman, 2019; Frömer, Lin, Dean Wolf, Inzlicht, & Shenhav, 2021; Limbachia et al., 2021). Other, more general, mathematical measures quantify the sense of control in terms of abstract, task-independent features of the environment (Klyubin, Polani, & Nehaniv, 2005; Ligneul, 2021; Huys & Dayan, 2009). While these measures have become increasingly popular in psychology (Goddu & Gopnik, 2024; Sandbrink & Summerfield, 2024) and machine learning (Leibfried, Pascual-Díaz, & Grau-Moya, 2019; Bharadhwaj, Babaeizadeh, Erhan, & Levine, 2022), the relationship between operational and mathematical measures, and their conceptual interpretations, remains unclear. Here, we unify these diverse perspectives on the sense of control within a coherent normative framework.

Theoretical Results

We consider agents (e.g., humans or animals) that interact with their environment by performing actions (e.g., a), which causally transition the environment from its current state s to a subsequent state s'. We assume that (i) agents know (or believe they know) the true dynamics of their environment (i.e., how states evolve), and (ii) there are no *a priori* extrinsic rewards or costs associated with environmental states. Our objective is to propose a measure that captures the agent's sense of control in a given state s. We begin with the intuitive notion that the sense of control in state s corresponds to the degree of *a priori* readiness for immediately collecting rewards or avoiding punishments, no matter what extrinsic reward function is subsequently announced. Informally, this implies that agents sense maximal control over their environment when they can immediately achieve whatever they desire. Formally, we assume that, at some point in time, agents are informed of a goal state g, which is randomly chosen and *a priori* unknown. The idea is thus to ask: 'In which state should one attempt to remain to be best prepared to reach g in a single step once it is announced?'

The answer to this question fundamentally depends on which reward statistics the agent seeks to maximize (i.e., its normative objective; Figure 1A1). We prove (not shown) that feeling prepared according to different objectives leads to preferences for conceptually distinct state features: (i) the number of available *distinct* actions (A-Control), (ii) the number of *certainly* reachable next states ($\mathrm{C_S}$-Control), or (iii) the number of *possibly* reachable next states ($\mathrm{P_S}$-Control). Moreover, we show that previously proposed measures of the sense of control exclusively capture only one of these conceptualizations of control, and, critically, different measures do not correspond to the *same* conceptualization. To resolve this discrepancy, we derive the degree-ℓ empowerment Emp_ℓ as a unified measure encompassing these different conceptualizations. While Emp_ℓ is derived purely from an abstract, top-down perspective, it admits a straightforward interpretation (Figure 1A2): It quantifies the sense of control in state s as how easily, on average, the agent can transition to other states in the environment. The empowerment's degree ℓ modulates the emphasis between *certain* ($\ell > 1$) and *possible* reachability ($\ell < 1$), where $\ell = 1$ corresponds exactly to counting *distinct* actions and is equivalent to the maximum expected reward (proof not shown).

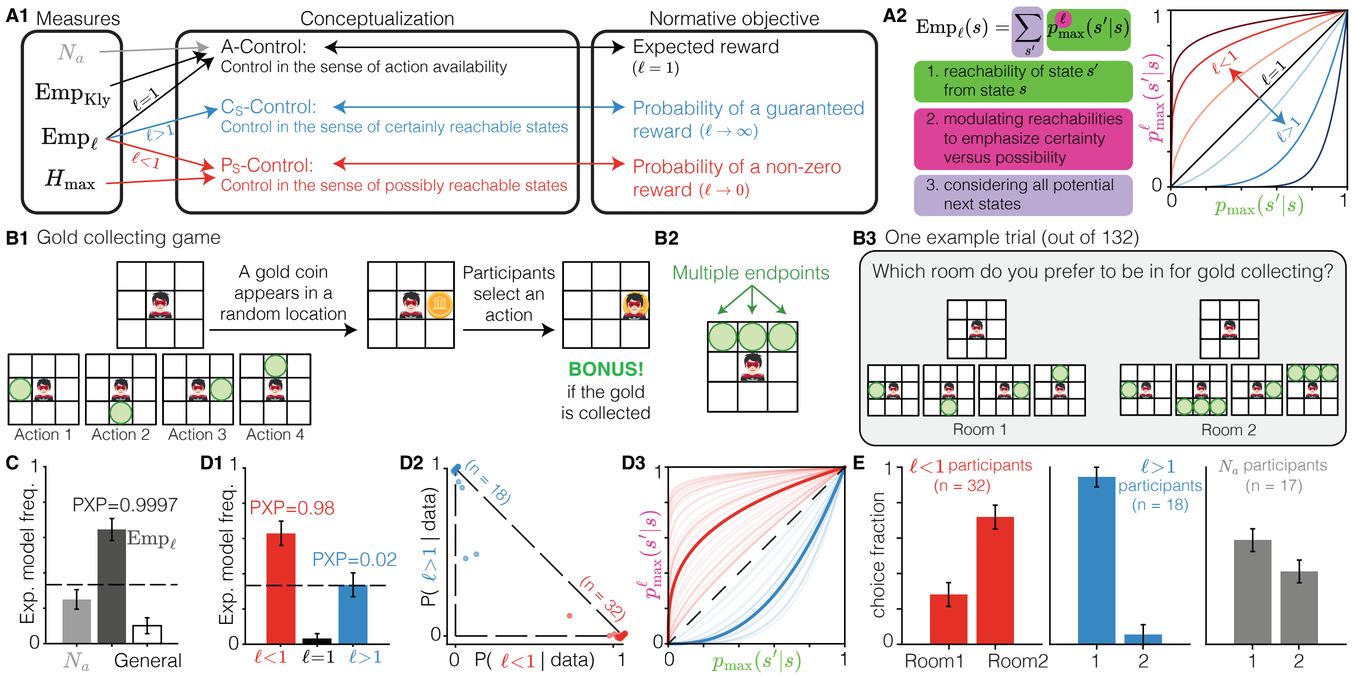

Figure 1: **A**. Different normative objectives correspond to different conceptualizations and existing measures of control (A1). Emp_ℓ unifies these conceptualization as in the decomposition (A2). The reachability of s' from s is defined as the maximum (over possible actions) probability of transitioning to s' from s, i.e., $p_{\max}(s'|s) := \max_a p(s'|s,a)$. N_a: the mere number of action; $\mathrm{Emp}_{\mathrm{Kly}}$: Klyubin empowerment (Klyubin et al., 2005) (= maximal transfer entropy; Ligneul, 2021); $H_{\max}$: Maximum outcome entropy (Huys & Dayan, 2009). **B**. Participants were first introduced to a gold-collecting game (B1). Action stochasticity was manipulated by adding multiple potential endpoints (B2). After the initial introduction, participants had to choose which room they would prefer for future gold collecting (knowing that the experimenter would then select the best of the actions available in that room; B3). **C**. General model selection (n=71). PXP: Protected Exceedance Probability (Rigoux et al., 2014). **D**. Emp_ℓ participants (n=50). D1. Model selection for ℓ. D2. Model probability simplex for individual participants (different data points). D3. Individual differences in the fitted value of ℓ (similar to A2). Thick curves: group-averaged. **E**. Fraction of choices of different participant groups for Room 1 versus Room 2 in B3. Error bars: The standard error of the mean.

Experimental Results

To determine whether Emp_ℓ explains the human sense of control, we designed a gold-collecting game involving 3×3 virtual rooms that had different numbers of available actions and different levels of outcome-stochasticity per action (Figure 1B1-B2). A gold coin would appear in one of the peripheral states (with equal probability) and might be potentially collectable by one of the actions (Figure 1B1). On each of the 132 trials, participants had to choose which of two rooms they preferred for this purpose (Figure 1B3). The experiment included 12 rooms whose properties were selected such that participants' action choices allowed us to identify their preferences for, e.g., certainty versus possibility (Room 1 vs. Room 2 in Figure 1B3), as confirmed through model- and parameter-recovery analyses (not shown). At the end of the experiment, participants completed the Locus-of-Control (Levenson, 1974) and Intolerance-to-Uncertainty (Carleton, Norton, & Asmundson, 2007) surveys.

The choices of the majority of 71 human participants were best explained by Emp_ℓ (n=50; with a normalized accuracy rate $>$ 80%), compared to the simple count of actions N_a (n=17) and a general model treating the sense of control in each of the 12 rooms as a separate free parameter (n=4; Figure 1C). Among participants best described by Emp_ℓ (Figure 1D), $^2/_3$ exhibited a preference for possible reachability ($\ell < 1$) and $^1/_3$ for certain reachability ($\ell > 1$). Notably, we did not find any participant purely seeking action availability ($\ell = 1$); this was despite the success of model-recovery in dissociating $\ell = 1$ from $\ell \neq 1$ (not shown) and was consistent with additional qualitative checks (Figure 1E). Finally, we studied the relationship between individual variability in fitted ℓ values and survey responses (32 questions). We found that the second principal component (PC) of the survey scores (out of four recoverable PCs identified via bootstrapping; explaining 11% of variance) predicted the fitted $\log \ell$ well ($\rho = 0.45$; $p = 0.002$; $BF = 23$; not shown). Bootstrapped loadings revealed that a high self-reported internal locus of control was associated with lower fitted ℓ values; however, examining this relationship in depth will require more data.

Conclusion

Our results unify and extend existing perspectives on the human sense of control, providing a foundation for capturing and characterizing individual differences in sensing control.

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