Suboptimal human decision-making reflects an efficient information bottleneck on inference

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Abstract

Human decision-making behavior varies widely across individuals and task conditions. This variability is often interpreted as a variety of suboptimal inference strategies, but the principles that govern these suboptimalities are not well understood. We propose that one major source of variability in suboptimal decision-making reflects a specific form of bounded rationality that involves capacity-limited inference. We developed and used new theoretical and empirical approaches to study capacity-limited inference based on the information-bottleneck framework. These approaches allowed us to relate the amount of information used (capacity), to the effectiveness with which it was used (accuracy), by individual human subjects performing a variety of inference tasks. We found that substantial variability both within and across subjects reflected optimal capacity-accuracy trade-offs. Strikingly, the same capacity-accuracy tradeoffs were evident among those using heuristic (biased) inference strategies, which inherently failed to maximize performance for a given level of information use but nonetheless appeared to be implemented in a similarly capacity-limited manner. The results imply that human inference reflects consequential, and flexible, capacity limitations that impose structure on suboptimal choice behavior.

Keywords: information bottleneck; inference; heuristics; individual variability; noise; decision-making; bounded rationality

Background

Many decisions depend on inferences about latent properties of the environment that determine the outcomes associated with different choices. These inferences require evaluating observations that provide probabilistic evidence for or against different possible latent states. For tasks requiring such inferences, human choice behavior varies widely between individuals (Glaze, Filipowicz, Kable, Balasubramanian, & Gold, 2018). This variability often manifests as a range of seemingly suboptimal strategies that differ in their degree of departure from optimal Bayesian inference. This high degree of individual variability poses a substantial challenge to elucidating the general mechanisms subserving inference in the brain.

One possible explanation for this range of suboptimal behaviors is bounded rationality (Simon, 1955). This idea proposes that decision-makers strive to perform effectively, but have limited resources such as time, energy, and computational capacity that are costly to use. Decision-makers can flexibly adjust the relative importance of limiting resource use versus maximizing performance, creating a natural stratification of strategies that range from maximally accurate but resource intensive, to resource-frugal but less accurate.

We propose that one form of bounded rationality arises from the costs associated with encoding information from the observations and using them to form inferences about the world. Accordingly, we predict that decision-makers flexibly adjust how much information their inferences retain from the observations, leading to an information bottleneck in the inference process. As detailed below, this idea predicts particular relationships between capacity and accuracy for different forms of suboptimal inference that we used to better understand individual differences in decision-making behaviors.

Information bottleneck framework

We evaluated these hypotheses by applying the **information bottleneck (IB)** method (Tishby, Pereira, & Bialek, 2000) to human inference. Let X be the discrete observations available to a decision-maker, Y be the latent states that are probabilistically related to the observations, and R be the potentially stochastic inferences a decision-maker makes about Y after observing X. We can measure how much information a decision-maker uses from the observations (**inference capacity**) with $I(X;R)$, the mutual information between X and R. Likewise, we can measure how closely their inferences match the true latent state (**inference predictiveness**) by $I(R;Y)$, the mutual information between R and Y. We can explicitly capture the tradeoff between information compression and accuracy with the IB optimization problem,

$$\min_{p(r|x)} I(X;R) - \beta I(R;Y), \tag{1}$$

where β determines the relative importance of inference capacity and inference predictiveness, and the problem is solved by optimizing the choice probabilities, $p(r|x)$. Solving (1) over the range of all β values results in the **IB bound**, which defines the maximum possible $I(R;Y)$ for a given $I(X;R)$. Given that we can compute $I(X;R)$ and $I(R;Y)$ from empirical choice data, we can evaluate our hypotheses by determining where individuals fall relative to this bound. Individuals that fall on or near the IB bound are **information efficient**.

Notably, under conditions common to many decision-making tasks, we find that solutions to (1) satisfy the following:

$$p^*(r=i|x) = \frac{e^{\beta\alpha^* p(y=i|x)}}{\sum_k e^{\beta\alpha^* p(y=k|x)}}, \tag{2}$$

where $p(y=i|x)$ is the posterior probability of the latent state corresponding to the inference $r=i$, β is the same β from (1), and α^* is the log of the accuracy-inaccuracy ratio for a given β. In other words, capacity-limited, information-efficient inference is equivalent to noisy optimal inference such that degree of compression is equivalent to noise magnitude.

Individual variability reflects optimal capacity-performance tradeoffs

We analyzed human choice behavior from two inference tasks. In one, subjects predicted which of two horses would win a race upon observing a set of shapes sampled based on which horse would win. In the second, subjects inferred from which of two hidden source jars beads were being drawn.

In all experiments, inference capacity varied substantially across individuals, but many were on the IB bound (**Figure 1**). However, the proportion of subjects on the bound varied by experiment. The choice behavior of individuals on the IB bound matched our theoretical result that capacity-limited,

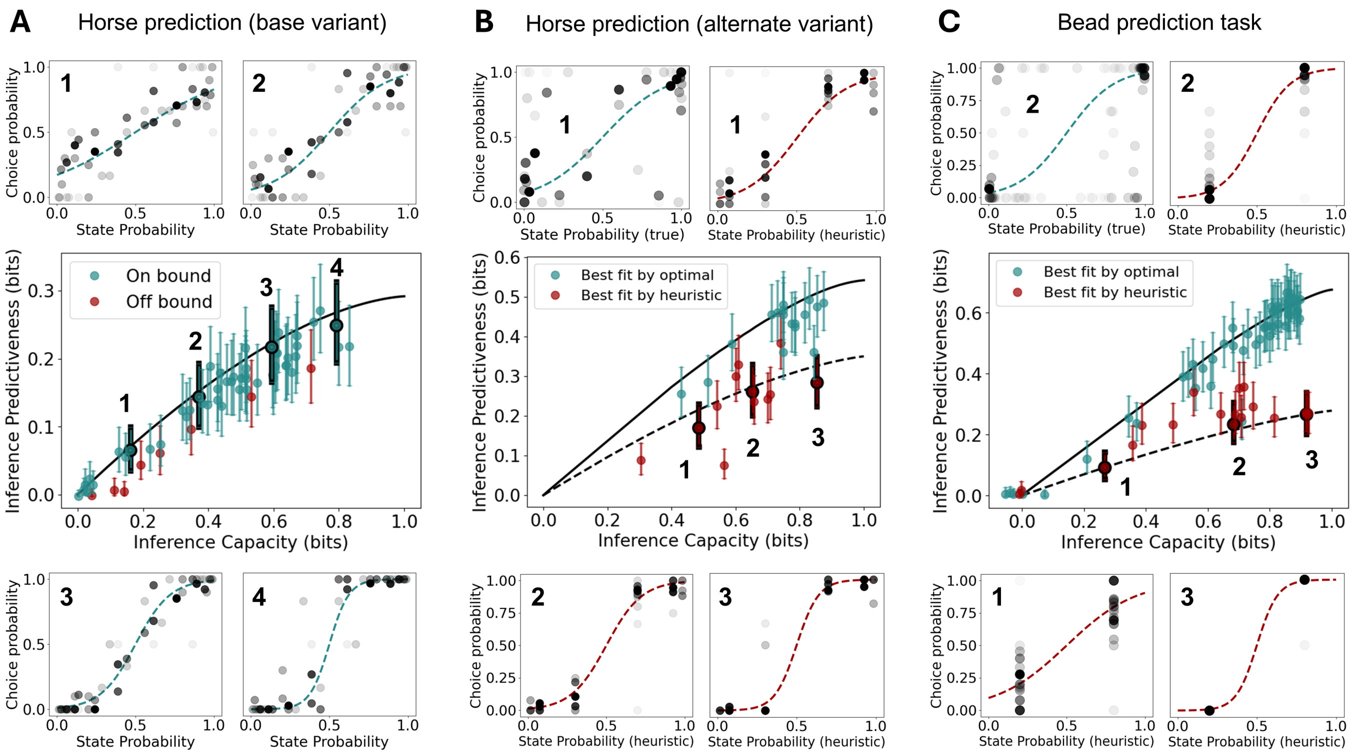

Figure 1: **A.** Individual subjects (1-4) on the IB bound (solid black curve, center plot) exhibit choice probabilities (black points, top/bottom plots) indicative of noisy optimal inference, matching theoretical predictions of IB-optimal behavior (dashed blue curves). Choice noise magnitude is tightly linked to inference capacity, decreasing 1 to 4. **B-C.** Many individuals off the IB bound fall along heuristic IB curves (dashed black curve). Example subjects (1-3) poorly match noisy optimal inference (top left plots) and instead match noisy heuristic inference (dashed red curves) with the same noise-capacity equivalence as the full IB bound.

information-efficient inference is equivalent to optimal inference corrupted by logistic choice noise. The empirical choice probabilities of these individuals (**Figure 1A**, top/bottom plots, black points) closely matched the theoretical choice probabilities computed using equation (2) (dashed blue curves), with inference capacity inversely related to choice noise magnitude (decreasing from 1 to 4) as predicted.

Any deviation from equation (2) falls off the bound. Thus, it is notable that many who fall off the bound exhibit choice behavior consistent with heuristic inference strategies. These simplify the inference problem by, for example, ignoring less informative observations or treating different observations as equally informative. The choice probabilities of these subjects poorly matched noisy optimal inference (**Figure 1B-C**, top-left plots), but closely matched noisy versions of task-specific heuristics (dashed red curves). Remarkably, the choice noise magnitude of these subjects varied with inference capacity in precisely the same way as those along the full IB bound. We confirmed these results with a standard model-based analysis, which showed these subjects were better fit by the noisy heuristic model (**Figure 1B-C**, center plots).

In a variant of the horse prediction task in which we manipulated decision time (**Figure 2**), we found that people increased their inference capacity when given more time to decide ($p <$1e-10). Crucially, people also increased their accuracy in a way that maintained information efficiency ($p <$1e-6).

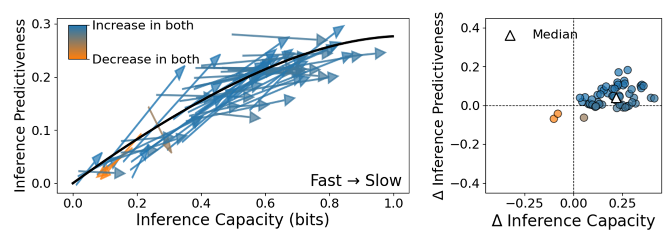

Figure 2: Horse prediction task, speed-accuracy variant.

In conclusion, we find optimal capacity-performance tradeoffs that are indicative of an information bottleneck acting on inference. Our approach also indirectly captures another major axis of individual variability - the use of heuristic strategies.

Acknowledgments

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