Small-Worlds Memory Generation and Capacity in the Neuroidal Model

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Abstract

Understanding higher level cognitive processes is a central problem in neuroscience. The Neuroidal model provides a useful framework for posing these problems in a computer science context. There has been significant recent work trying to understand memory capacity in the Neuroidal model but this work was done assuming that the network of neurons was an Erdős-Rényi (ER) random graph. However, the network of neurons in the brain has been shown to exhibit small-world properties, which are not present in ER graphs. In this research we explore replacing ER graphs with Watts-Strogatz and Barabási-Albert small-worlds graphs in order to more accurately model the biological reality. We aim to investigate the implications for memory capacity and interference within the Neuroidal model. We show that the algorithm JOIN can function with small-worlds graph structures and allow the Neuroidal model to reach capacity.

Keywords: neuroidal computation; unsupervised learning; small-worlds; lifelong learning

Introduction

We build upon the Neuroidal model to show that its underlying graph structure can contain small-worlds subgraphs. We study its ability to memorize information with neuronal structures that carry more biological plausibility within a computational context. We offer novel results on showing the viability of two types of small-worlds graph structures for unsupervised memorization. We utilize an empirical simulation framework to observe memory capacity. These advancements open up possibilities for further work on neurobiologically-grounded learning across a lifetime.

Background

The Neuroidal model is an algorithmic, deterministic, spiking neural network with uniformly random synaptic connectivity, hetero-associative memory, and with weak and synchronized timing mechanisms (Valiant, 2000). The Neuroidal model is a directed, Erdős–Rényi (ER) G_{np} random graph that encodes memories. It consists of a number of neurons n, edge probability p, and with synapse weights assigned $\frac{1}{k}$.

The JOIN algorithm has been introduced to perform memory generation by connecting two existing memories to a newly created memory. JOIN is reminiscent of *coding* within the brain, which has been found to occur in real neural systems (Tacikowski et al., 2024). Extensive results have been found for JOIN's effect on the overall capacity of the model when neuron sharing is allowed (Perrine et al., 2024).

Small-worlds Background

Since its inception by Milgram (1967), many neuroscience studies have investigated the structure of random brain networks and found that they exhibit small-world properties (Bassett & Bullmore, 2006; Zippo et al., 2013). It has also been shown that real-world brain networks show power-law degree distributions, which are also not present in ER graphs (Chialvo, 2004; Tomasi et al., 2017).

Watts-Strogatz (WS) Graphs WS graphs are expansions of ER graphs, where each node is first connected to d nearest neighbors, then every edge is considered with the probability p to be rewired or replaced. When p is low, nodes remain connected to their nearest neighbors. When p is high, it becomes increasingly similar to an ER graph (Watts & Strogatz, 1998).

Barabási-Albert (BA) Graphs BA graphs are first initialized with d nodes, each with a connection to every other node. At each step, a node is added to the graph, which then samples d nodes from the existing network and considers a connection to each with a probability p_i, which differs based on the degree of the outgoing node (Albert & Barabási, 2002).

Graph Metrics

Here are three key measurements that are commonly used to describe the small-worlds properties of a graph:

- **Average Path Length:** mean of all shortest path lengths.
- **Clustering Coefficient:** connectedness of subgraphs.
  - *Global:* ratio of all triangles to all triples in the graph.
  - *Local:* ratio of existing edges between neighbors to the total possible incident edges.

A graph is said to be a "small-world" when it has both a *low average path length* and a *high clustering coefficient*.

Methodology

For optimization purposes, we offer the following adjacency matrix equations to calculate average path length, the global clustering coefficient, and the local clustering coefficient:

$$L = \frac{1}{n \cdot (n-1)} \sum_{i \neq j} min\{A^1_{ij}, \dots, A^k_{ij}\}. \tag{1}$$

$$C = \frac{\sum_{i,j,h} A_{ij} A_{jk} A_{ki}}{\sum_i k_i \cdot (k_i - 1)}. \tag{2}$$

$$C_i = \frac{\sum_{j,h} A_{ij} A_{ih} A_{jh}}{k_i \cdot (k_i - 1)}. \tag{3}$$

We perform memorization within two specific instances of the Neuroidal model, as derived from previous work (Valiant, 2005). Our code is available here: `https://github.com/chandradeep24/Valiant/tree/adjacency`

Results

To validate our simulation's small-world properties, we ran comparisons between several additional graph metrics and highlight the most notable results here. Figures 1 and 2 show averaged results from over 100 trials where $n = 10^5$. These graph metrics suggest that BA graphs should be effective for the Neuroidal model, due to their consistency. This perception changes when we perform memorization to capacity.

Average Path Length Figure 1 shows that ER and WS graphs had difficulty connecting to every node, causing infinite average path lengths. The path length of BA graphs remained consistent, making them seemingly the most desirable.

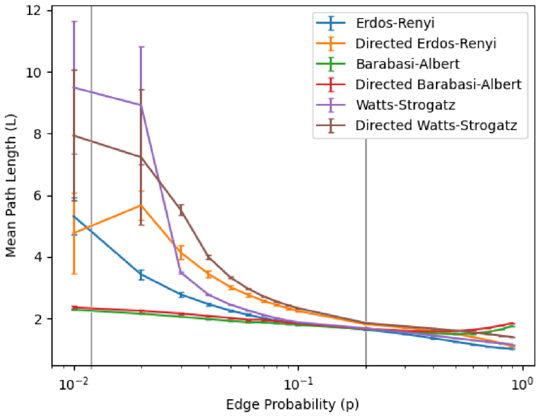

Figure 1: Edge Probability (p) vs. Mean Path Length (L)

Clustering Coefficients Figure 2 shows calculation of the global clustering coefficient. BA graphs still do the best at maintaining a higher clustering coefficient within sparse graphs. Results for the local coefficient are omitted, as their charts did not reveal novel behavior.

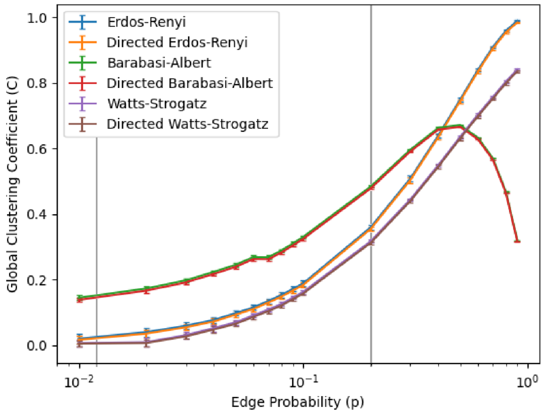

Figure 2: Probability (p) vs. Global Clustering Coefficient (C)

Capacity Results

For each capacity result in Table 1, we have our Neuroidal model simulation attempt to memorize approximately 500,000 items of information until an interference threshold of 1% is reached. Each simulation was run 10 times on a consumer personal computer, and then the results were averaged.

$n = 10^4$, $d = 2^7$, $k = 2^5$	ER	WS	BA
Time (sec)	296	52	560
Capacity	973	255	4
Interference	1.08%	1.25%	1.2%

$n = 10^4$, $d = 2^{11}$, $k = 2^6$	ER	WS	BA
Time (sec)	41	63	578
Capacity	226	238	4
Interference	1.08%	1.21%	1.2%

Table 1: Graph Type Capacity Simulation Results

The JOIN algorithm is shown to continue to perform well within small neighborhoods, as with ER and WS graphs. Surprisingly, Barabási-Albert graphs performed the worst here, likely a direct result of the large hubs causing a high interference rate. BA graphs showed promise for stability and clustering, however this was found to be detrimental to the model.

Conclusion

We have shown that Watts-Strogatz and Barabási-Albert graphs can be substituted for Erdős-Rényi graphs in the Neuroidal model. Our results show that BA graphs may not be sufficient for use within the Neuroidal model, at least within the tested model specifications.

Previous work shows evidence that ER graphs could be sufficient for modeling memorization in the hippocampus (Perrine et al., 2024). Further work could include investigation of quantitative parameters between a different region, such as the entorhinal cortex (Tacikowski et al., 2024), in comparison to our small-worlds Neuroidal model.

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